ISSN:0975-9646

Suraj Singh Panwar et al, / (IJCSIT) International Journal of Computer Science and Information Technologies, Vol. 5 (1) , 2014, 112-118

New Mandelbrot and Julia Sets for Transcendental Function

Suraj Singh Panwar[#1], Mr.Pawan Kumar Mishra[*2]

[#] *M.Tech. - CSE, Scholar, Faculty of Technology,*
Computer Science and Engineering Department,
Uttarakhand Technical University, Dehradun, INDIA

[*] *Assistant Proff., M.Tech. CSE,Faculty of Technology,*
Computer Science and Engineering Department,
Uttarakhand Technical University, Dehradun, INDIA

***Abstract*— In this paper we consider the dynamics of complex transcendental function i.e. cos function using iterative procedure known as Ishikawa Method. Fractals are generated and analyzed for integer and non-integer values. New Mandelbrot and Julia sets are generated for different values of parameters defined by Ishikawa iteration and transcendental controlled function. Here different ovoid's or lobe are analyzed and our study relies on the generation of mini Mandelbrot and Julia sets.**

***Keywords*— Mandelbrot Set, Julia Sets, Complex, Fractals, Ishikawa iteration, Transcendental function, Ovoids.**

I. INTRODUCTION

Complex Graphics of nonlinear dynamical systems have been a focus of research nowadays. These graphics of complex plane is studied under Fractal Theory. A Fractal is a statistical shape that is difficult and detailed at every level of magnification, as fit as self-similar. Fractal is defined as a set, which is self–similar under magnification [1]. Self-similarity means looking the same structure over all ranges of scale, i.e. a small section of a fractal can be viewed as a part of the larger fractal. Fractal Theory is an exciting branch of applicable Mathematics and Computer Science. Benoit Mandelbrot (1924-2010) is known as the father of fractal geometry. He coined the word fractal in the late 1970s. He explain geometric fractals as "a rough or fragmented geometric shape that can be divided into parts, every one of which is a reduced-size duplicate of the whole".[2] There are many variety of fractals found in nature in the form of many usual objects such as mountains, coastlines, trees ferns and clouds[3,4,5]. They all are fractals in nature and can be represented on a computer by a recursive algorithm of computer graphics.

The Julia sets and the Mandelbrot sets are two most important images under various researches in the field of fractal theory [6]. In 1918, French Mathematician Gaston Julia (1893-1978) [7] investigated the iteration process of a complex function and attained a Julia set, whereas the Mandelbrot set was given by Benoit B. Mandelbrot [2] in 1979.

II. PRELIMINARIES AND DEFINATION

Here we have used the transformation function $Z \rightarrow (Z^n+C)$, $n>=2.0$ and $C=\cos(1/\#pixel^p)$, $p>=1.0$ for generating fractal images with respect to Ishikawa iterates, where z and c are the complex quantities and n, p are real numbers. Each of these fractal images is constructed as a two-dimensional array of pixels. Each pixel is represented by a pair of (x, y) coordinates. The complex quantities z and c can be represented as:

$$Z=Z_x+iZ_y$$

$$C=C_x+iC_y$$

where $i=\sqrt{(-1)}$ and Z_x , C_x are the real parts and Z_y & C_y are the imaginary parts of Z and C , respectively. The pixel coordinates (x, y) may be associated with (C_x , C_y) or (Z_x , Z_y). Based on this concept, the fractal images can be classified as follows:

z-Plane fractals, where in (x,y) is a function of (z_x,z_y).

c-Plane fractals, where in (x,y) is a function of (c_x,c_y).

In the literature, the fractals for n=2 in z plane are termed as the Mandelbrot set while the fractals for n=2 in c plane are known as Julia sets [8].

A. Mann's Iteration : One Step Iteration

Mann's iteration technique is a one-step iteration technique given by William Robert Mann (1920-2006), a mathematician from Chapel Hill, North Carolina. The iteration technique involves one step for iteration, and is given as [9,10]:

$$x_{n+1} = s.f(x_n) + (1-s)x_n, \text{ where } n\geq 0 \text{ and } 0 < s < 1$$

B. Ishikawa Iteration : Two Step Iteration

Ishikawa iteration [11,12] technique is a two-step iteration method known after Ishikawa. Let X be a subset of complex number and $f : X \to X$ for all $x_0 \in X$, we have the sequence numbers for $\{x_n\}$ and $\{y_n\}$ in X according to following way. [13] :

$$y_n = S'_n f(x_n) + (1 - S'_n) x_n$$

$$x_{n+1} = S_n f(y_n) + (1 - S_n) x_n$$

Where, $0 \leq S'_n \leq 1$, $0 \leq S_n \leq 1$ and S'_n & S_n are both convergent to non-zero number.

III. Generation Of Relative Superior Mandelbrot Sets

We present here some Relative Superior Mandelbrot sets[14,15] for the function Z→ (Z^n+C), n>=2.0 and C=cos(1/#pixel^p), p>=1.0 for integer and some non-integer values of n and p. Process of generating fractal images is similar to self-squared function [16]. The Fractals have been generated by the iterative procedure starting with initial value z_0, where z and c are both complex quantities. The parameter s and s' also changes the structure and beauty of fractals.

A. Relative Superior Mandelbrot sets for Quadratic function:

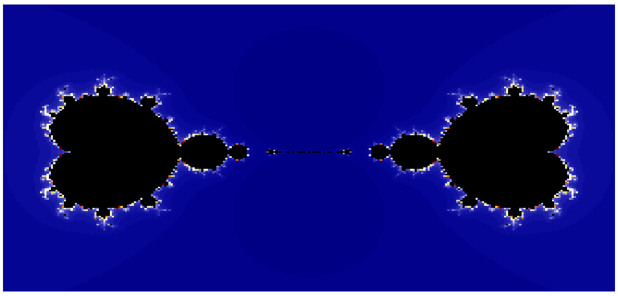

Fig.1 Relative Superior Mandelbrot set for s=s'=1,p=1,n=2.

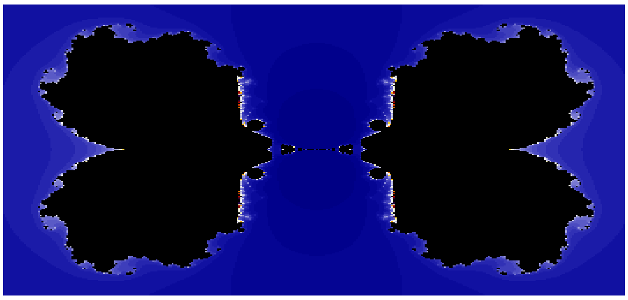

Fig. 2 Relative Superior Mandelbrot set for s=s'=0.5,p=1,n=2.

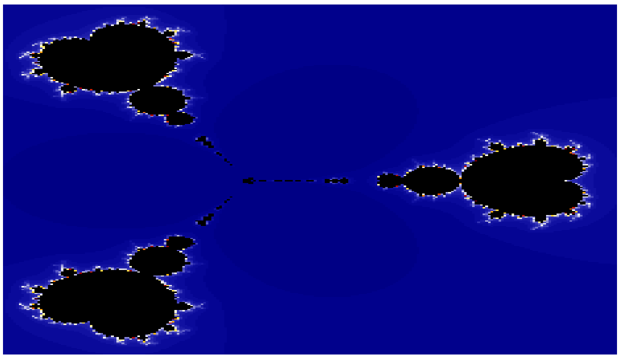

Fig. 3 Relative Superior Mandelbrot set for s=s'=1,p=1.5,n=2.

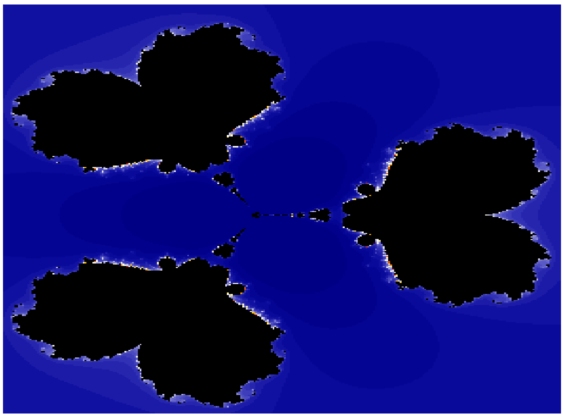

Fig. 4 Relative Superior Mandelbrot set for s=s'=0.5,p=1.5,n=2

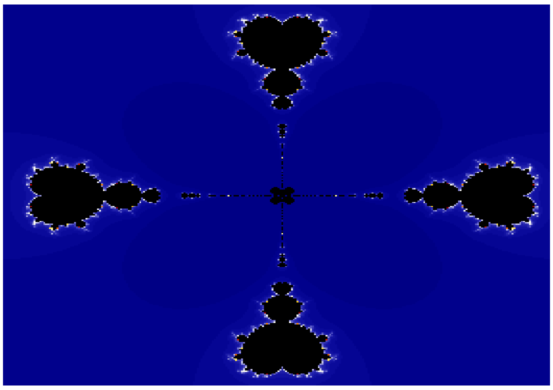

Fig. 5 Relative Superior Mandelbrot set for s=s'=1,p=2,n=2

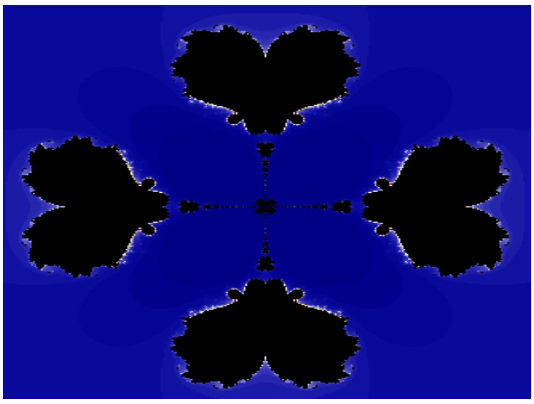

Fig. 6 Relative Superior Mandelbrot set for s=s'=0.5,p=2,n=2

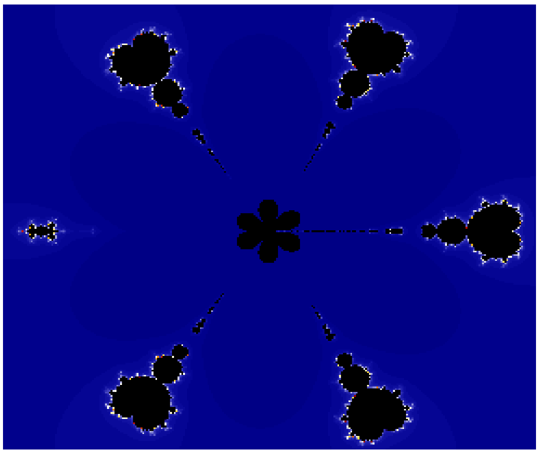

Fig. 7 Relative Superior Mandelbrot set for s=s'=1,p=2.9,n=2

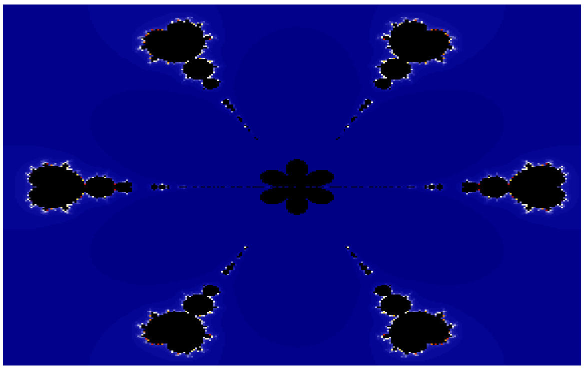

Fig. 8 Relative Superior Mandelbrot set for s=s'=1,p=3,n=2

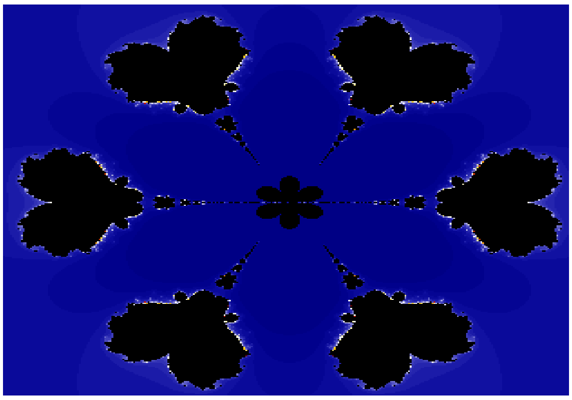

Fig. 9 Relative Superior Mandelbrot set for s=s'=0.5,p=3,n=2

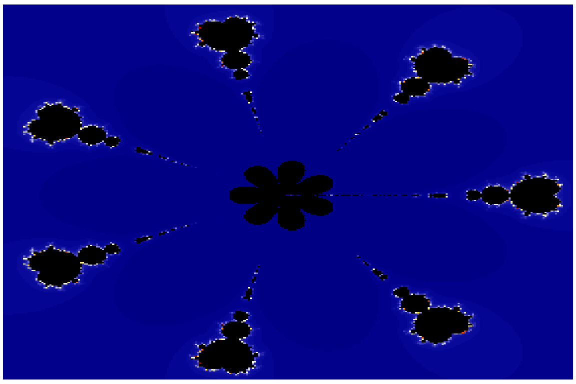

Fig. 10 Relative Superior Mandelbrot set for s=s'=1,p=3.5,n=2

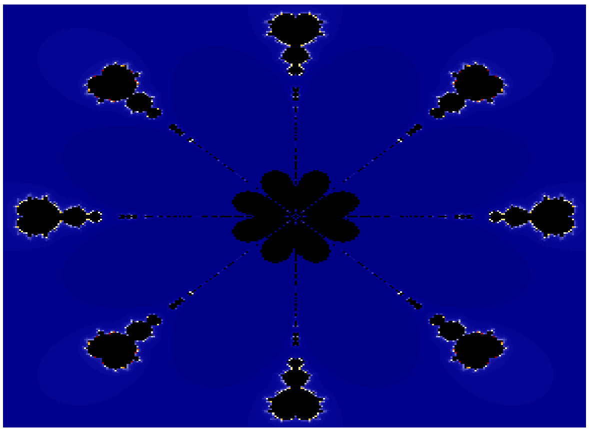

Fig. 11 Relative Superior Mandelbrot set for s=s'=1,p=4,n=2

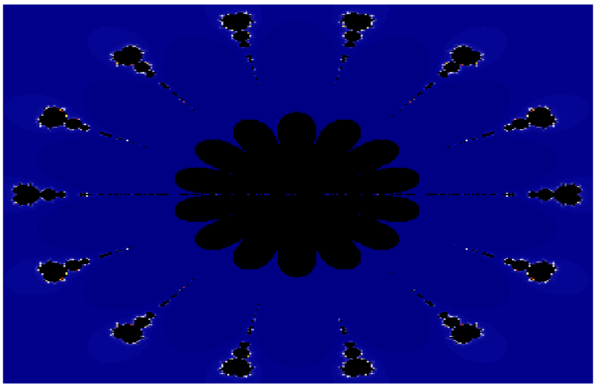

Fig. 12 Relative Superior Mandelbrot set for s=s'=1,p=7,n=2

B. ***Relative Superior Mandelbrot set for Cubic function.***

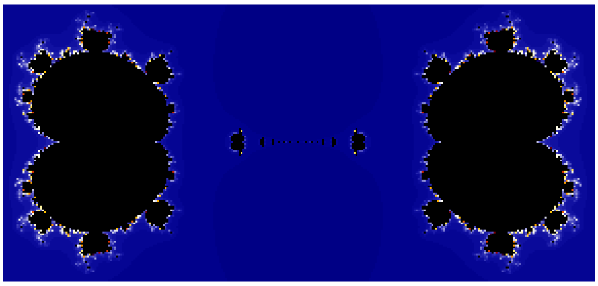

Fig. 13 Relative Superior Mandelbrot set for s=s'=1,p=1,n=3.

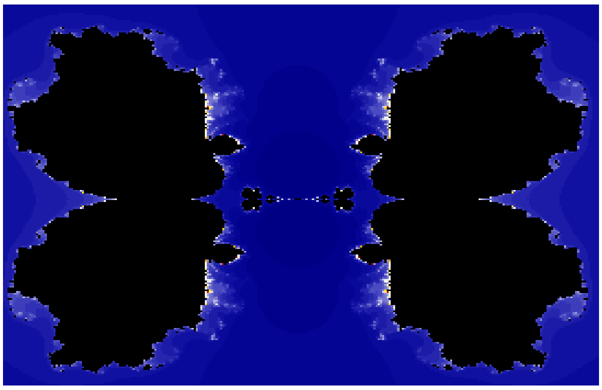

Fig. 14 Relative Superior Mandelbrot set for s=s'=0.5,p=1,n=3.

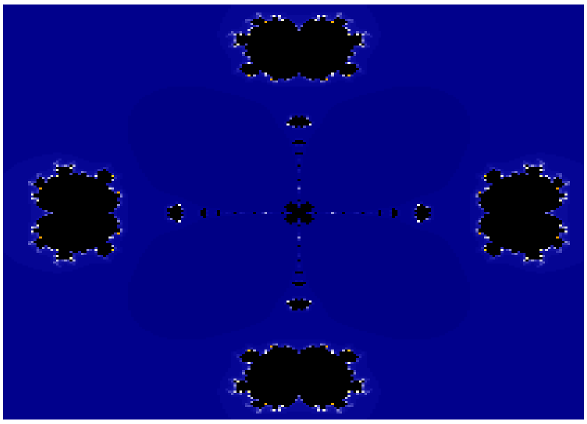

Fig. 15 Relative Superior Mandelbrot set for s=s'=1,p=2,n=3.

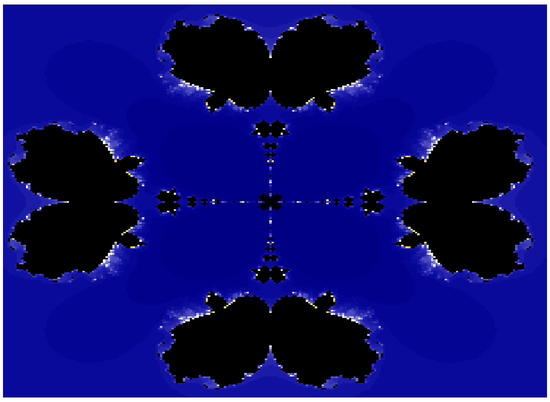

Fig. 16 Relative Superior Mandelbrot set for s=s'=0.5,p=2,n=3.

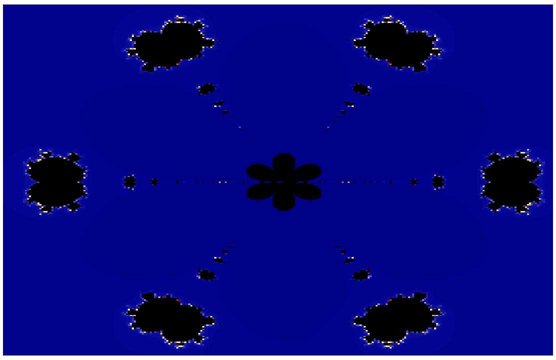

Fig. 17 Relative Superior Mandelbrot set for s=s'=1,p=3,n=3.

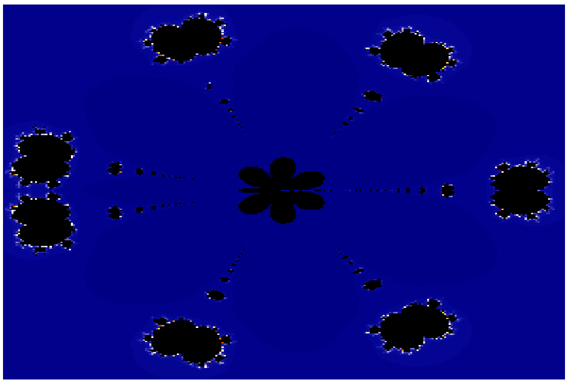

Fig. 18 Relative Superior Mandelbrot set for s=s'=1,p=3.2,n=3.

C. Relative Superior Mandelbrot set for Bi-Quadratic function.

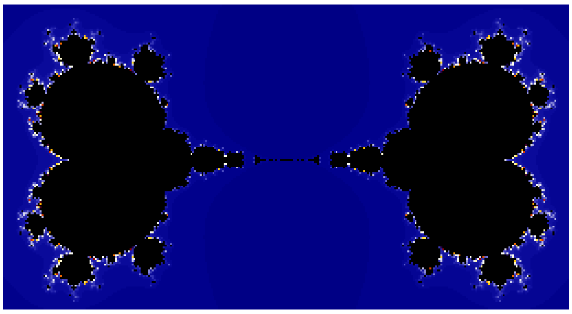

Fig. 19 Relative Superior Mandelbrot set for s=s'=1,p=1,n=4

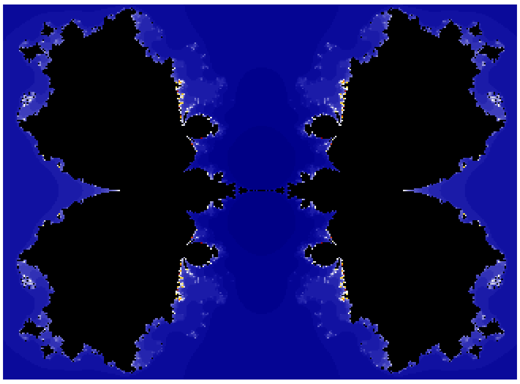

Fig. 20 Relative Superior Mandelbrot set for s=s'=0.5,p=1,n=4

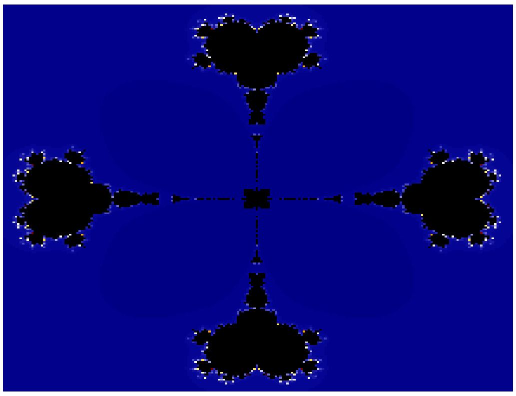

Fig. 21 Relative Superior Mandelbrot set for s=s'=1,p=2,n=4

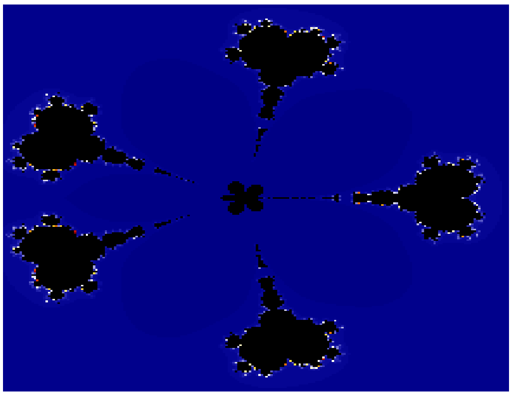

Fig. 22 Relative Superior Mandelbrot set for s=s'=1,p=2.3,n=4

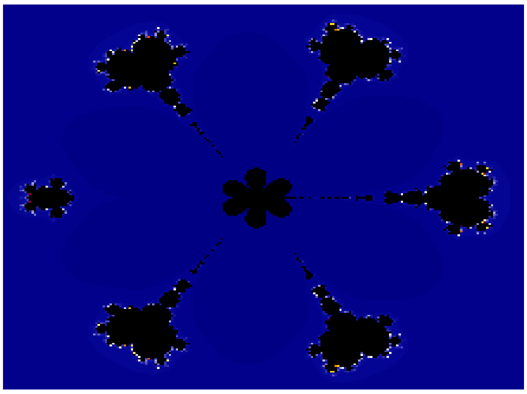

Fig. 23 Relative Superior Mandelbrot set for s=s'=1,p=2.9,n=4

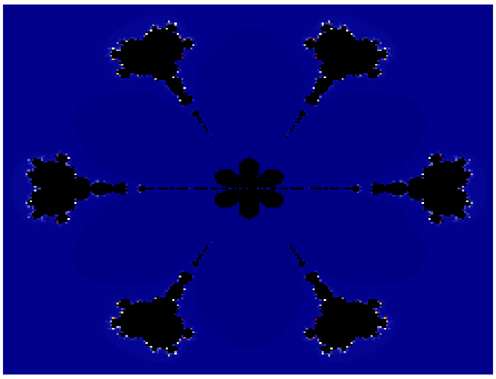

Fig. 24 Relative Superior Mandelbrot set for s=s'=1,p=3,n=4

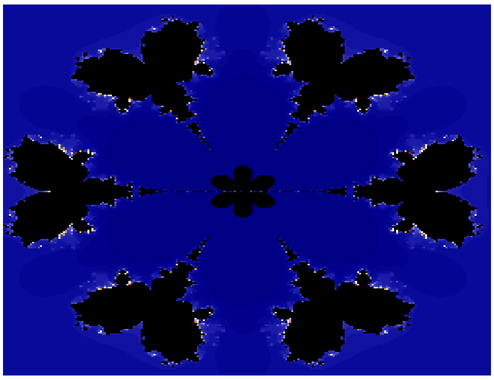

Fig. 25 Relative Superior Mandelbrot set for s=s'=0.5,p=3,n=4

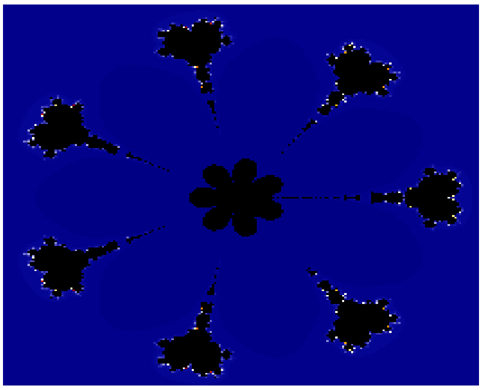

Fig. 26 Relative Superior Mandelbrot set for s=s'=1,p=3.5,n=4

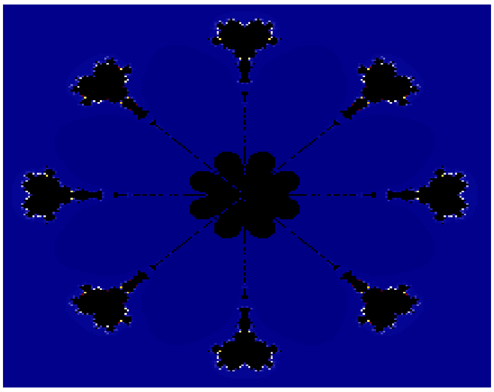

Fig. 27 Relative Superior Mandelbrot set for s=s'=1,p=4,n=4

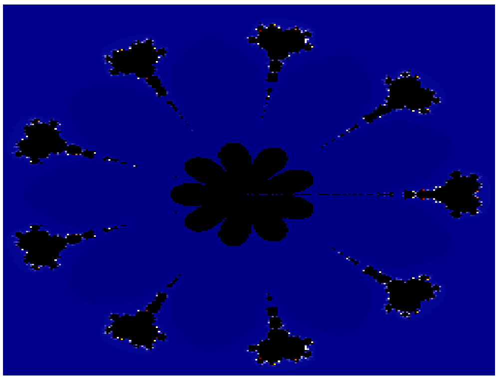

Fig. 28 Relative Superior Mandelbrot set for s=s'=1,p=4.5,n=4

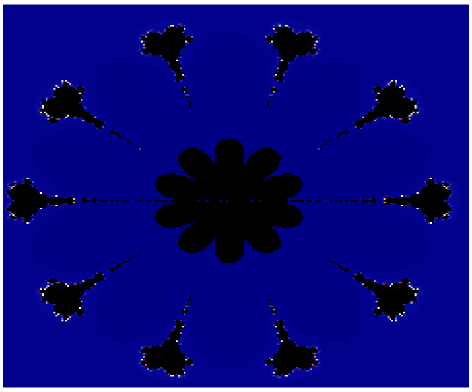

Fig. 29 Relative Superior Mandelbrot set for s=s'=1,p=5,n=4

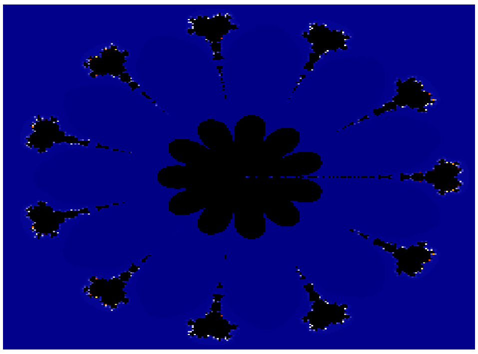

Fig. 30 Relative Superior Mandelbrot set for s=s'=1,p=5.5,n=4

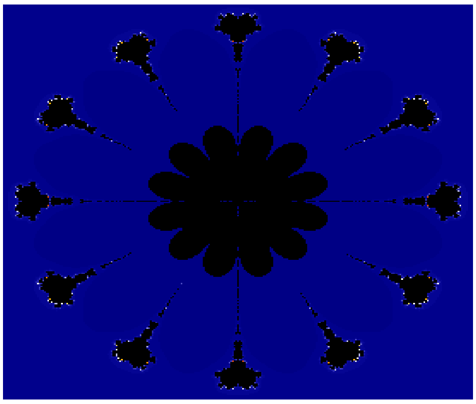

Fig. 31 Relative Superior Mandelbrot set for s=s'=1,p=6,n=4

IV. GENERATION OF RELATIVE SUPERIOR JULIA SETS

We present here some Relative Superior Julia sets for the function Z→ (Z^n+C), n>=2.0, C=cos(1/#pixel^p), p>=1.0 for integer and some non-integer values. The parameter s and s' also changes the structure and beauty of fractals. Following are some of the Julia sets for different values of p and n for quadratic, cubic and bi-quadratic functions.

A. *Relative Superior Julia Sets for Quadratic function.*

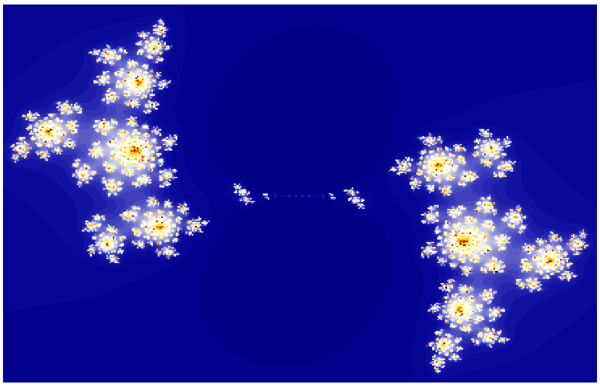

Fig. 32 RSJS for s=s'=1, p=1, n=2, c= 0.03125 + 0.65625i

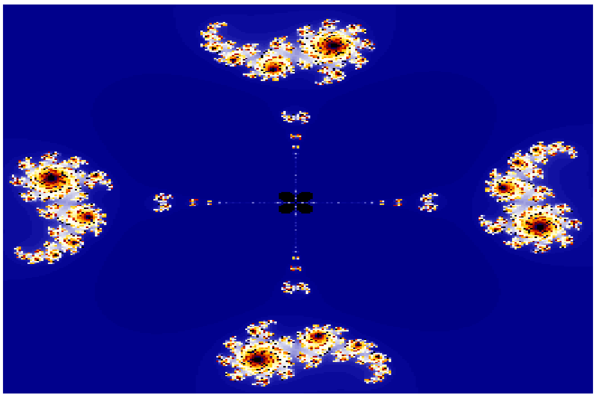

Fig. 33 RSJS for s=s'=1, p=2, n=2, c= 0.375+ -0.159i

B. *Relative Superior Julia sets for Cubic function.*

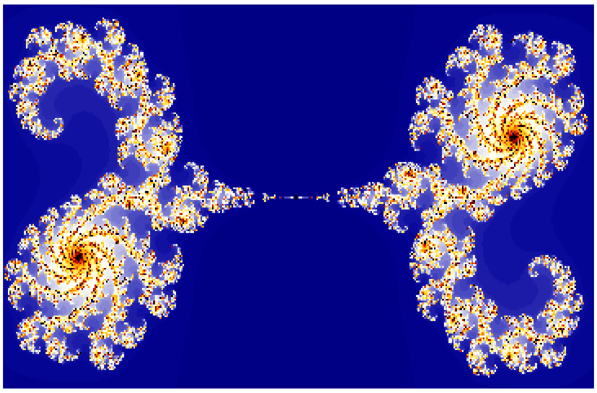

Fig. 34 RSJS for s=s'=1, p=1, n=3, c= 0.4583 + 0.0347i

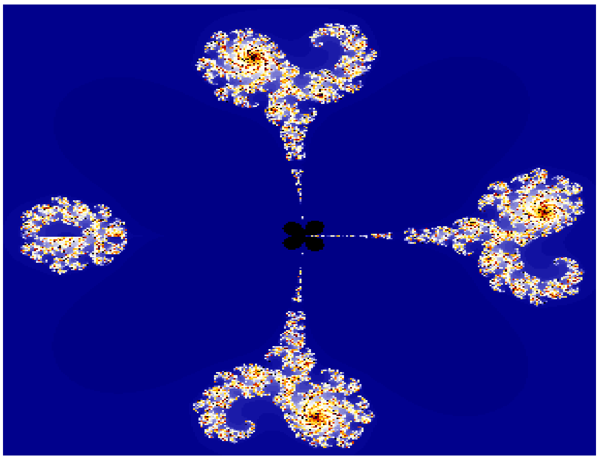

Fig. 35 RSJS for s=s'=1, p=1.9, n=3, c= 0.4583 + 0.0347i

C. *Relative Superior Julia sets for Bi-Quadratic function.*

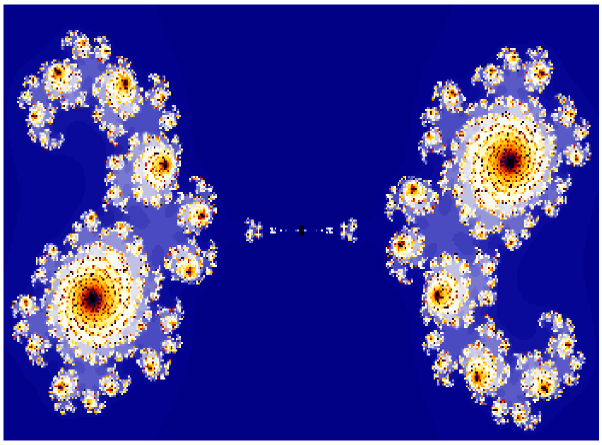

Fig. 36 RSJS for s=s'=1, p=1, n=4,c= 0.5764 + 0.0695i

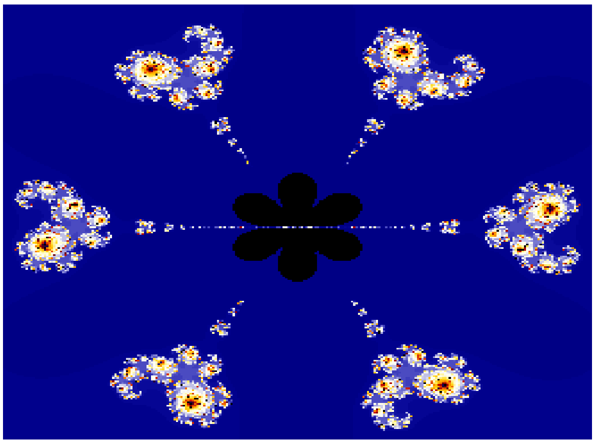

Fig. 37 RSJS for s=s'=1, p=3, n=4 c= 0.5764 + 0.0695i

V. CONCLUSION

In the complex dynamics polynomial function for z→ (z^n+c), where n>=2.0, control transcendental function is c=cos(1/#pixel^P), p>=1.0. The fractals generated with power p and n are found as rotationally symmetric. We have analysis superior iterates at different power of n and p as shown in fig. There are ovoids or bulbs attached with main body. The number of major secondary lobe is (n-1).

The controlling function cos for value c exhibits new characteristics for the generating fractals. Here we have presented the geometric properties of fractals along different axis. The fractals generated depends on the parameter p. From above observation and analysis 2p image of mini Mandelbrot is generated for above cos controlling function. For p=1, two (2p) mirror image (similar faced) of mini Mandelbrot bulbs are obtained at an angle of 180^0. For p=2, four mirror image of mini Mandelbrot bulbs at an angle of $360^0/4=90^0$ are separated from each other. So for integer value of p there are 2p mirror image separated at an angle of 360^0/2p. For non-integer value of p two new images are created from left side step by step at p+0.2 and p+0.9 till the upper integer value is met.

For integer values of p and n, Mandelbrot Sets are symmetrical along x and y-axis, while for non-integer values of p, fractals are symmetrical only along x-axis. We have also generated some superior function controlled Julia sets using cos function, which resemble to beautiful images, full of colorful spiral, symmetrical and other shapes. Further by using different iterative methods and more different functions we can create fractals and analyse their properties.

REFERENCES

[1] B. B. Mandelbrot,*" Fractals and Chaos"* Berlin: Springer. pp. 38, ISBN 978-0-387-20158-0. "A fractal set is one for which the fractal (Hausdorff-Besicovitch) dimension strictly exceeds the topological dimension", 2004.

[2] B. B. Mandelbrot, "*The fractal geometry of nature.*" Macmillan. ISBN 978-0-7167-1186-5, 1983.

[3] M. F. Barnsley, R. L. Devaney, B. B. Mandelbrot, H. O. Peitgen, D. Saupe, and R. F. Voss, *"The Science of Fractal Images",* Springer – Verlag 1988.

[4] M. Batty, *"Fractals - Geometry Between Dimensions,"* New Scientist (Holborn Publishing Group) 105 (1450): 31, 1985-04-04.

[5] H. Peitgen and P. H. Richter, *"The Beauty of Fractals",* Springer-Verlag, Berlin, 1986.

[6] G. Edgar, *"Classics on Fractals",* Boulder, CO: Westview Press. ISBN 978-0-8133-4153-8, 2004.

[7] G. Julia, " Sur I' iteration des functions rationnelles", J.Math Pure Appli. 8 (1918), 737-747.

[8] K. W. Shirriff, *"An investigation of fractals generated by $z \rightarrow z^{-n} + c$ ",* Computers and Graphics, pp. 603-607, 13, 4,1993.

[9] A. Negi, and M. Rani, *"Midgets of Superior Mandelbrot Set",* Chaos, Solitons and Fractals, July 2006.

[10] S. Shukla and A. Negi," *Study of Newbie Fractal controlled by log function",* International Journal of Computer Technology and Applications (IJCTA), Vol 4 (2), pp.341-346.

[11] A. Negi, *"Generation of Fractals and Applications",* Thesis, Gurukul Kangri Vishwavidyalaya, 2005.

[12] R. Rana, Y. S. Chauhan and A. Negi, *"Ishikawa Iterates for Logarithmic function",* International Journal of Computer Applications (IJCA), Volume 15, No.5, February 2011.

[13] S. Ishikawa, *"Fixed points by a new iteration method",* Proc. Amer. Math. Soc.44, (1974), pp.147-150.

[14] M. Rani, and V. Kumar, *"Superior Mandelbrot Set",* J.Koreans Soc. Math. Edu. Ser. D (2004) 8(4), pp. 279-291.

[15] M. Rani, *"Iterative procedures in fractals and chaos",* Ph.D. Thesis, Department of Computer Science, Faculty of Technology, Gurukul Kangri Vishvavidyalaya, Hardwar, 2002.

[16] R. L. Daveney, *"An Introduction to Chaotic Dynamical Systems",* Springer-Verlag, New York. Inc.1994.